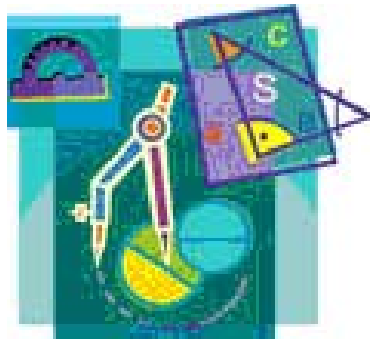


$$a^2 + b^2 = c^2$$

Teaching Math in the Multi-level Classroom

Part I: Focus on Geometry



Susan K. Pittman-Shetler

Bonnie Goonen

$$a^2 + b^2 = c^2$$

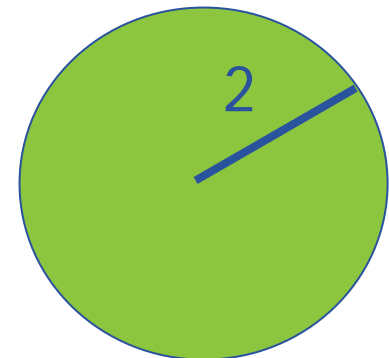
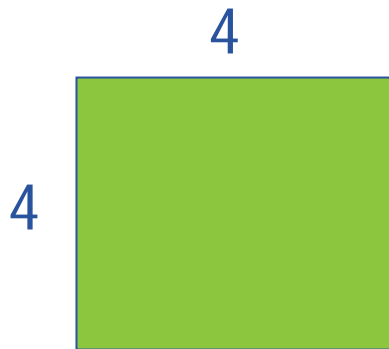
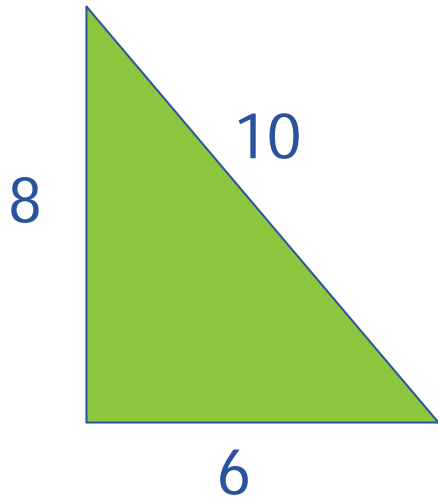
Focus on Geometry

Let's Review!

$$a^2 + b^2 = c^2$$

Puzzler: Exploring Patterns

What curious property do each of the following figures share?



Focus on Geometry

Geometry is the mathematics of shape and space. It's about the properties of objects (their angles and surfaces, for instance) and the consequences of how these objects are positioned (where their shadows fall, how people must move around them).

Focus on Geometry

Why Is Geometry Important?

- The world is built of shape and space, and geometry is its mathematics.
- Informal geometry is good preparation for more abstract geometric reasoning. Students have trouble with abstraction if they lack sufficient experience with more concrete materials and activities.
- Geometry has more applications than just within the field itself. Students can often solve problems from other fields more easily when they represent the problems geometrically.
- Because many people think well visually, Geometry can be a doorway to their success in mathematics.
- Geometry is important to real life.

Focus on Geometry

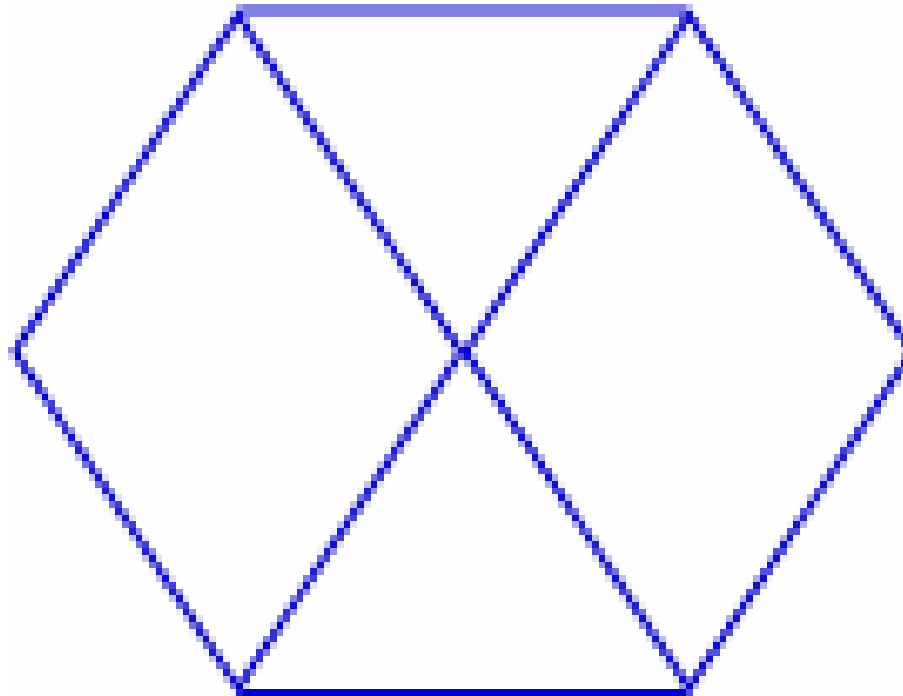
How Do You Use Geometry?

$$a^2 + b^2 = c^2$$



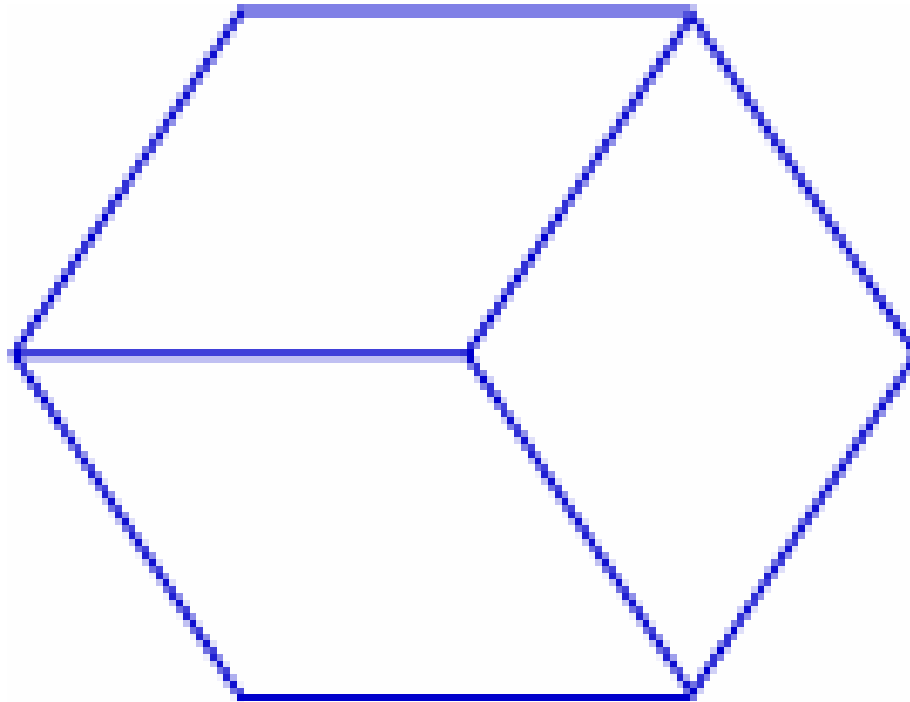
Visualizing
The First Step
in Geometric Reasoning

Visualizing in Geometry



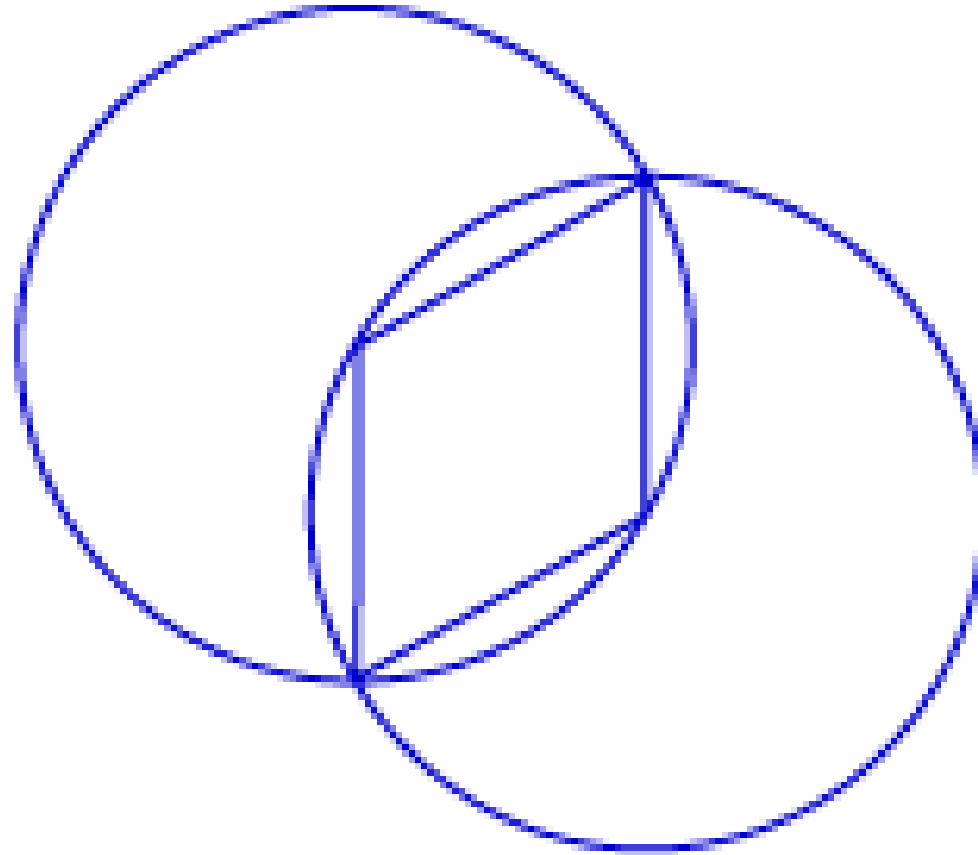
$$a^2 + b^2 = c^2$$

Visualizing in Geometry



$$a^2 + b^2 = c^2$$

Visualizing in Geometry



$$a^2 + b^2 = c^2$$

Visualizing in Geometry

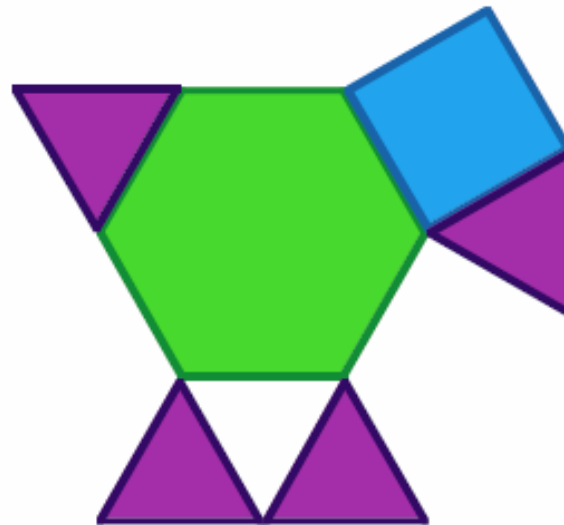
Description 1:

The design looks like a bird with

- a hexagon body;
- a square for the head;
- triangles for the beak and tail; and
- triangles for the feet.

Visualizing in Geometry

Description 1



$$a^2 + b^2 = c^2$$

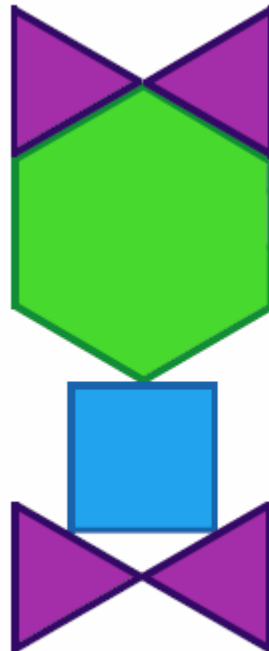
Visualizing in Geometry

Description 2:

- Start with a hexagon.
- On each of the three topmost sides of the hexagon, attach a triangle.
- On the bottom side of the hexagon, attach a square.
- Below the square, attach two more triangles with their vertices touching.

Visualizing in Geometry

Description 2



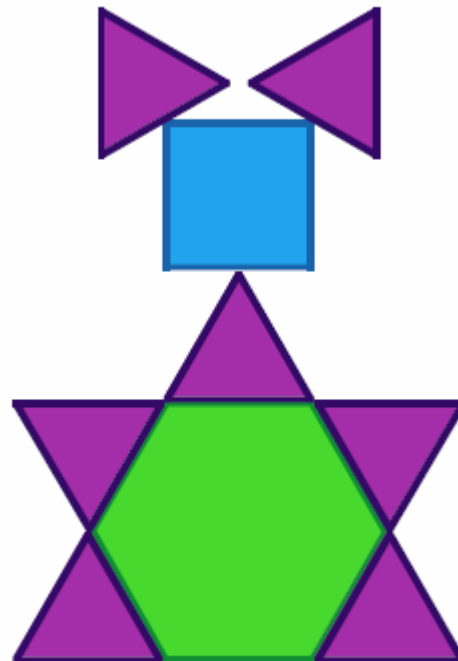
Visualizing in Geometry

Description 3:

- Start with a hexagon. Position it so that it has two horizontal sides.
- On each of the non-horizontal sides, attach a triangle so that the side of the triangle exactly matches the side of the hexagon.
- On the top side of the hexagon, attach a triangle so that the side of the triangle exactly matches the side of the hexagon.
- Take a square and place it above the top triangle. It should be placed so that the vertex of the triangle is at the midpoint of a side of the square and the sides of the square are horizontal and vertical.
- On each of the two top vertices of the square, attach a triangle. Place each triangle so that the vertex of the square is at the midpoint of the side of the triangle and the side of the triangle makes 45° angles with the two sides of the square it touches.

Visualizing in Geometry

Description 3



Make Sure We Have the Vocabulary!

It's Geometry Match!

$$a^2 + b^2 = c^2$$

Focus on Geometry

Skills Required by the GED Mathematics Test

- model and solve problems using the concepts of perpendicularity, parallelism, congruence, and similarity of geometric figures.
- use spatial visualization skills to describe and analyze geometric figures and translations/rotations of such figures.
- use the Pythagorean Theorem to model and solve problems.
- find, use, and interpret the slope of a line, the y -intercept of a line, and the intersection of two lines.
- use coordinates to design and describe geometric figures.
- identify and select appropriate units of metric and customary measures.
- convert and estimate units of metric and customary measure (all conversions within systems).
- solve and estimate solutions to problems involving length, perimeter, area, surface area, volume, angle measurement, capacity, weight, and mass.
- use uniform rates (e.g., miles per hour, bushels per acre) in problem situations.
- read and interpret scales, meters, and gauges.
- predict the impact of changes in linear dimension on the perimeter, area, and volume of figures.

Focus on Geometry

Teaching Geometry: A Few Strategies

- Have high expectations for all students.
- Base practice on educational research.
- Integrate content areas.
- Encourage cooperative learning and collaboration with others.
- Use technology as a tool.
- Use inquiry-based learning.
- Promote mathematical reasoning and problem-solving skills.
- Use hand-on activities to model topics.
- Cluster concepts.
- Reflect on learning.
- Integrate assessment and instruction.

Focus on Geometry

There is a story often told about the writer Gertrude Stein. As she lay on her deathbed, a brave friend leaned over and whispered to her, “Gertrude, what is the answer?” With all her strength, Stein lifted her head from the pillow and replied, “What is the question?” Then she died.

The Question Is . . .

- GEDTS Statistical Study for Mathematics
 - Results were obtained from three operational test forms.
 - Used the top 40% of the most frequently missed test items.
 - These items represented 40% of the total items on the test forms.
 - Study focused on those candidates who passed (410 standard score) +/- 1 SEM called the NEAR group (N=107,163), and those candidates whose standard scores were +/- 2 SEMs below passing called the BELOW group (N=10,003).

GEDTS Conference, July 2005

Most Missed Questions

- How are the questions distributed between the two halves of the test?
 - Total number of questions examined: 48
 - Total from Part I (calculator): 24
 - Total from Part II (no calculator): 24

Math Themes - Most Missed Questions

- Theme 1: Geometry and Measurement
- Theme 2: Applying Basic Math Principles to Calculation
- Theme 3: Reading and Interpreting Graphs and Tables

Most Missed Questions: Geometry and Measurement

Do the two groups most commonly select the same or different incorrect responses?

	Geometry
Same	13
Different	2

It's clear that both groups find the same questions to be most difficult and both groups are also prone to make the same primary errors.

Most Missed Questions: Geometry and Measurement

- Name the type of Geometry question that is most likely to be challenging for the candidates

The answer! The Pythagorean Theorem

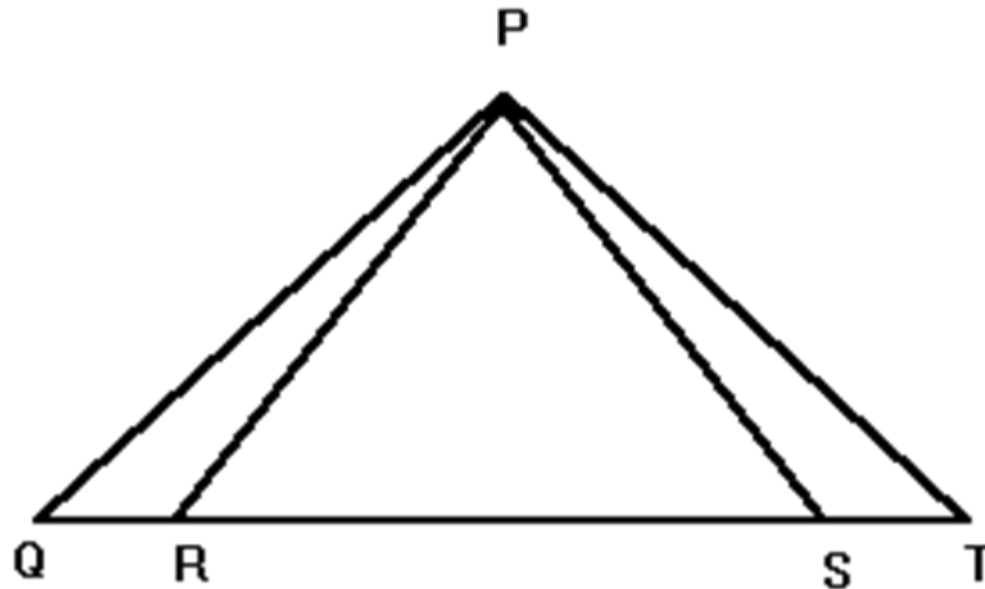
	Form #1	Form #2	Form #3
Found?	Yes	Yes	Yes
Difficult?	Yes	Yes	Yes

Most Missed Questions: Geometry and Measurement

- Pythagorean Theorem
- Area, perimeter, volume
 - Visualizing type of formula to be used
 - Comparing area, perimeter, and volume of figures
 - Partitioning of figures
 - Use of variables in a formula
- Parallel lines and angles

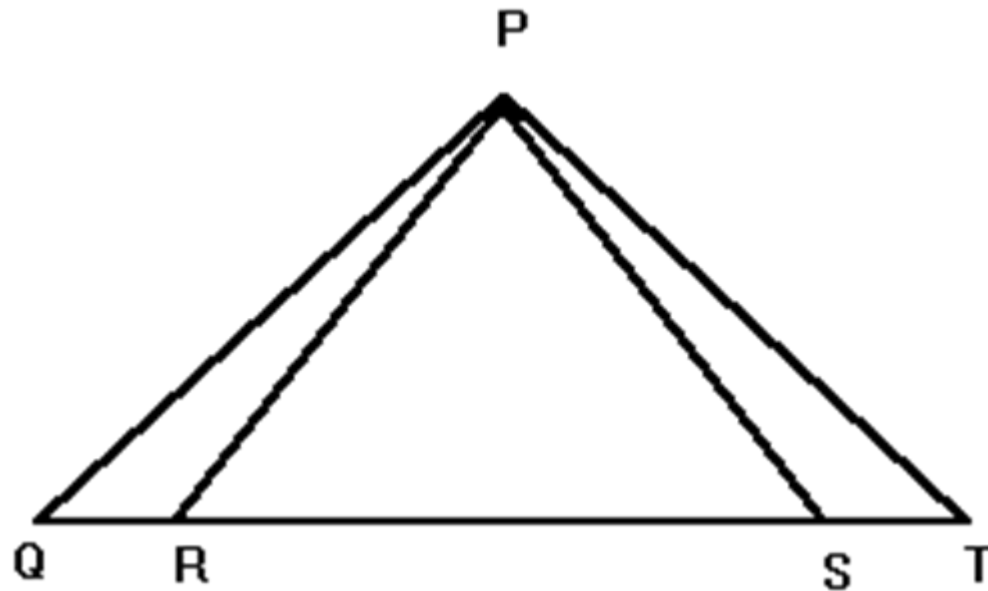
Getting Started with Geometry and Measurement!

- In the following diagram of the front view of the Great Pyramid, the measure of the angle PRQ is 120 degrees, the measure of the angle PQR is 24 degrees, and the measure of the angle PST is 110 degrees. What is the measure of the angle RPS in degrees?



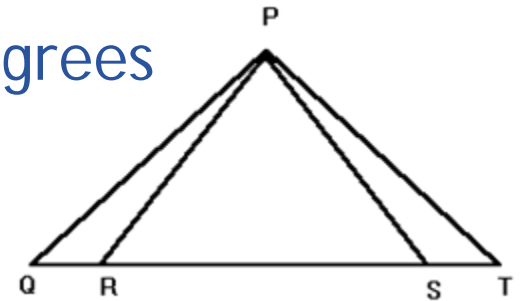
Getting Started with Geometry and Measurement!

- Hint:
 - How many degrees are there in a triangle or a straight line?



Answer

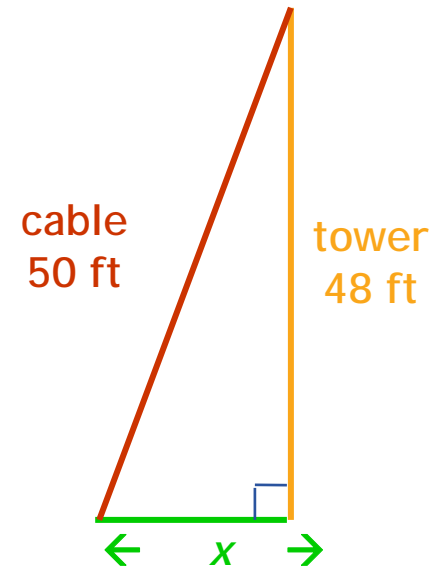
- $180 \text{ degrees} - 120 \text{ degrees} = 60 \text{ degrees}$
- $180 \text{ degrees} - 110 \text{ degrees} = 70 \text{ degrees}$
- $60 \text{ degrees} + 70 \text{ degrees} = 130 \text{ degrees}$
- $180 \text{ degrees} - 130 \text{ degrees} = 50 \text{ degrees}$



- In words, the problem would be as follows:
 - Angle PRQ = 120 degrees so Angle PRS has 60 degrees.
 - Angle PST has 110 degrees so Angle PSR has 70 degrees.
 - We know that the triangle PRS has 60 + 70 degrees in two of its angles to equal 130 degrees, therefore the third angle RPS is $180 - 130$ degrees or 50 degrees.

Most Missed Questions: Geometry and Measurement

One end of a 50-ft cable is attached to the top of a 48-ft tower. The other end of the cable is attached to the ground perpendicular to the base of the tower at a distance x feet from the base. What is the measure, in feet, of x ?



- (1) 2
- (2) 4
- (3) 7
- (4) 12
- (5) 14

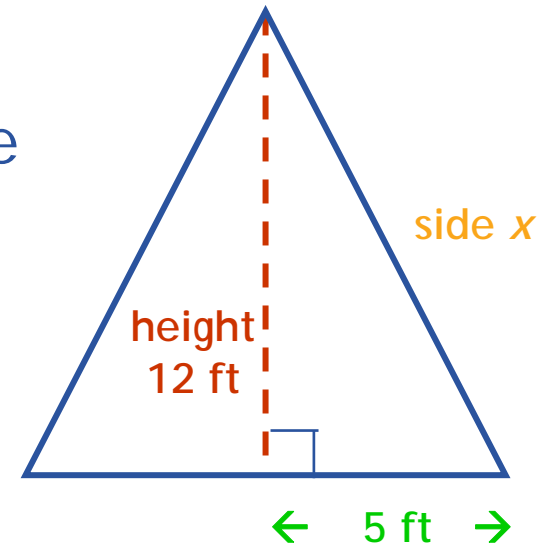
Which incorrect alternative would these candidates most likely have chosen?

(1) 2 **Why?**

The correct answer is (5): 14

Most Missed Questions: Geometry and Measurement

The height of an A-frame storage shed is 12 ft. The distance from the center of the floor to a side of the shed is 5 ft. What is the measure, in feet, of x ?



- (1) 13
- (2) 14
- (3) 15
- (4) 16
- (5) 17

Which incorrect alternative would these candidates most likely have chosen?

(5) 17 **Why?**

The correct answer is (1): 13

Most Missed Questions: Geometry and Measurement

- Were either of the incorrect alternatives in the last two questions even possible if triangles were formed?
- Theorem: The measure of any side of a triangle must be LESS THAN the sum of the measures of the other two sides. (This same concept forms the basis for other questions in the domain of Geometry.)

Focus on Geometry

Some say

it's all about

TRIANGLES!

$$a^2 + b^2 = c^2$$

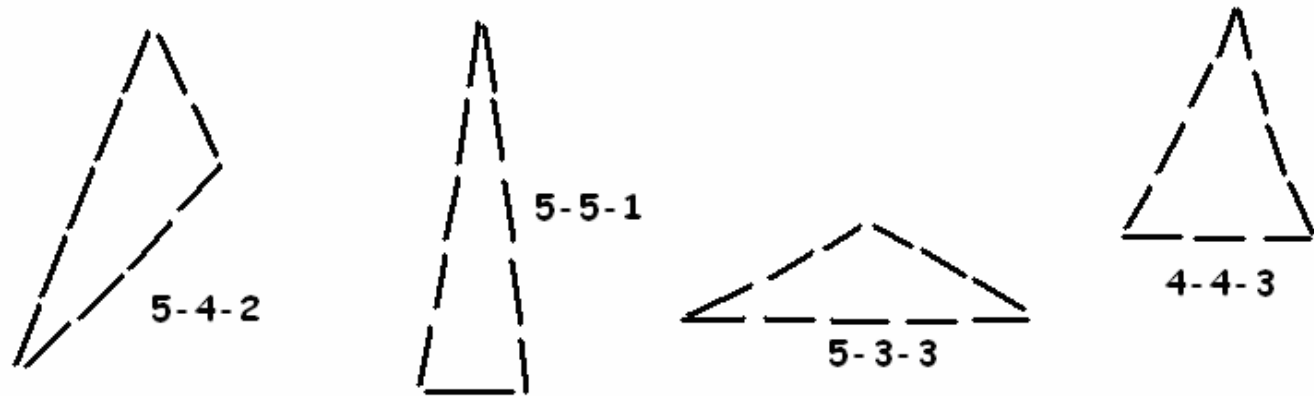
Exploring Triangles

How many different triangles can you make with 11 toothpicks?

(using a whole number of toothpicks per side and all 11 sticks for each triangle)

$$a^2 + b^2 = c^2$$

Exploring Triangles



Why isn't 6-3-2 a valid triangle?

Exploring Triangles

How many different triangles can you make with 12 toothpicks?

- *Surprisingly the count goes down if you add another matchstick to the mix: With 12 matchsticks one can only make three different triangles: 5-5-2; 5-4-3; 4-4-4.*
- *What's going on?*
- *When using this brainteaser, you may wish to have students make a table showing the number of integer triangles one can make with 1, 2, ..., 20 toothpicks. What do they notice about the even and odd entries? Do they see any patterns?*

Exploring Triangles

Problem A1

- Draw any triangle on your paper.
- Draw a second triangle that is different in some way from your first one. Describe in just a word or two how it is different.

Problem A2

- Draw a third triangle that is different from both of your other two. Describe how it is different.

Problem A3

- Draw two more triangles, different from all the ones that came before.

Problem A4

- To make "different" triangles, you have to change some feature of the triangle. Make a list of the features of triangles that you changed.

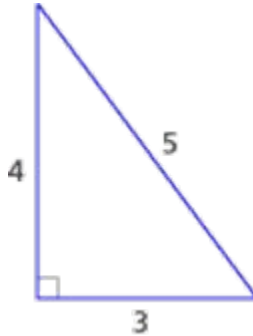
Exploring Triangles

More than one feature can be combined into a triangle. Decide which of the following combinations are possible. If the combination is possible, draw a sketch on a piece of paper. If not, explain why not.

- a. a scalene right triangle
- b. an isosceles right triangle
- c. an equilateral right triangle
- d. a right triangle that is also obtuse

Exploring Triangles

a. This is possible.

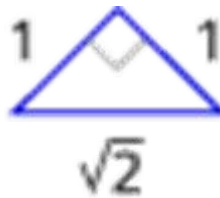


b. This is impossible. Inside a triangle, equal angles correspond to equal sides, so for a right triangle to be equilateral, it would have to have three right angles. Two right angles next to each other, however, form parallel lines, which would mean it would not be possible to complete such a triangle.



Exploring Triangles

c. This is possible.

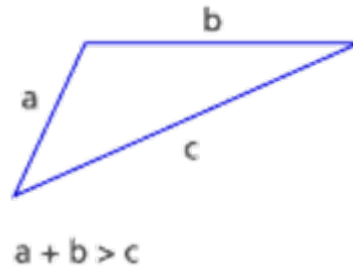


d. This is impossible. The picture below shows the right angle and obtuse angle next to each other, with the side in between laid out horizontally. The side that extends from the right angle is vertical, while the side that extends from the obtuse angle is pointed away from the side that extends from the right angle. Because these sides must be connected to form a triangle, this kind of triangle is impossible to make.



Exploring Triangles

- Theorem: The measure of any side of a triangle must be LESS THAN the sum of the measures of the other two sides. (This same concept forms the basis for other questions in the domain of Geometry.)



- The triangle inequality is the mathematical statement of the old adage, "The shortest distance between two points is a straight line." If you don't travel along the straight line, you travel two sides of a triangle, and that trip takes longer.

Most Missed Questions: Geometry and Measurement

Below are rectangles A and B with no text. For each, do you think that a question would be asked about area or perimeter?



A



B

A: Area Perimeter Either/both

Perimeter

B: Area Perimeter Either/both

Area

Time Out for Exploration

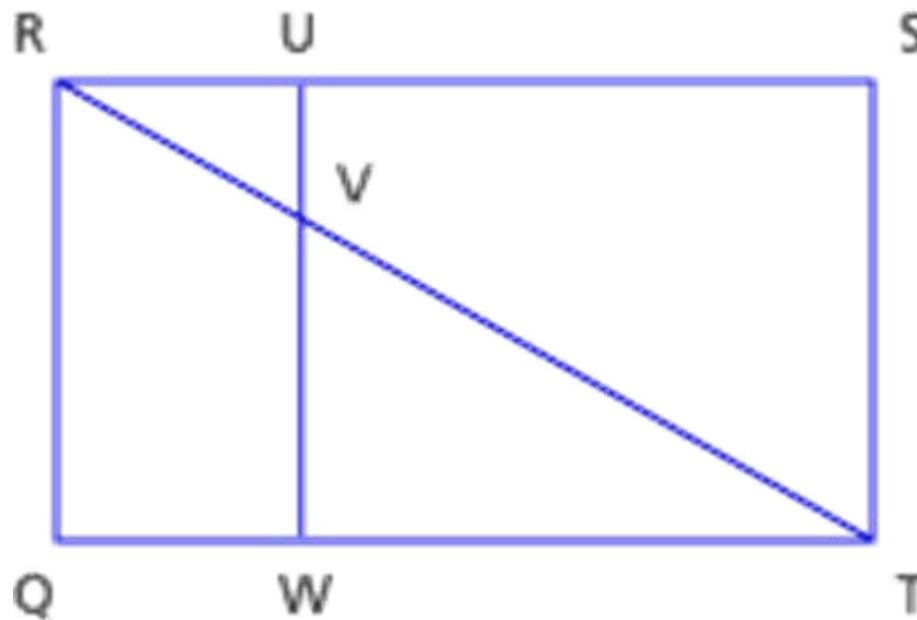
Polygons are two-dimensional geometric figures with these characteristics:

- They are made of straight line segments.
- Each segment touches exactly two other segments, one at each of its endpoints.
- They are closed -- they divide the plane into two distinct regions, one "inside" and the other "outside" the polygon.



Time Out for Exploration

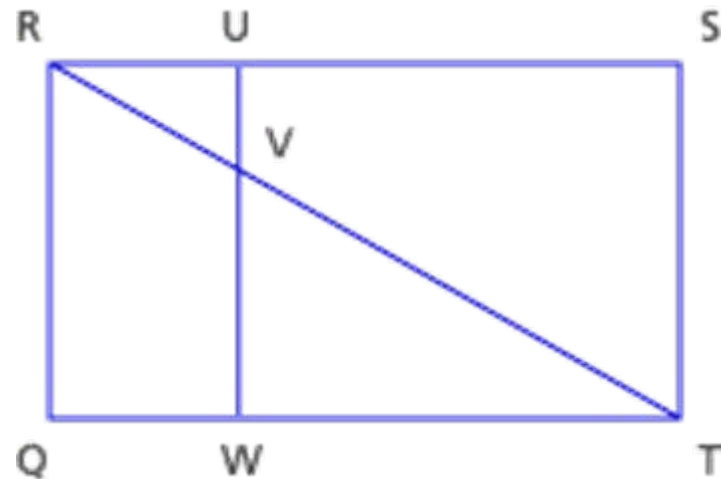
How many polygons can you find in the following figure?



Time Out for Exploration

There are 13 polygons. They are as follows:

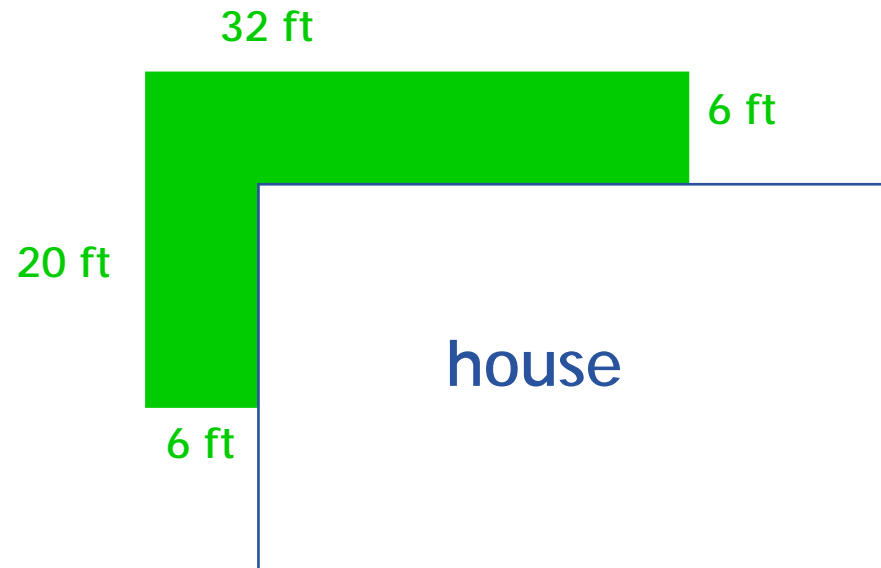
- Two small triangles (RUV and TWV)
- Their two complements (hexagons VUSTQR and VWQRST)
- Two quadrilaterals (RQWV and TSUV)
- Their two complements (pentagons VRSTW and VTQRU)
- Two larger triangles (RQT and TSR)
- Three rectangles (RUWQ, USTW, and QRST)



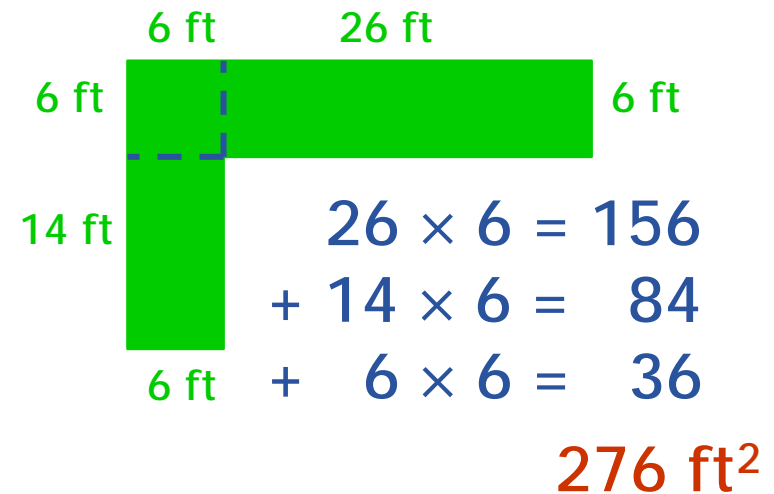
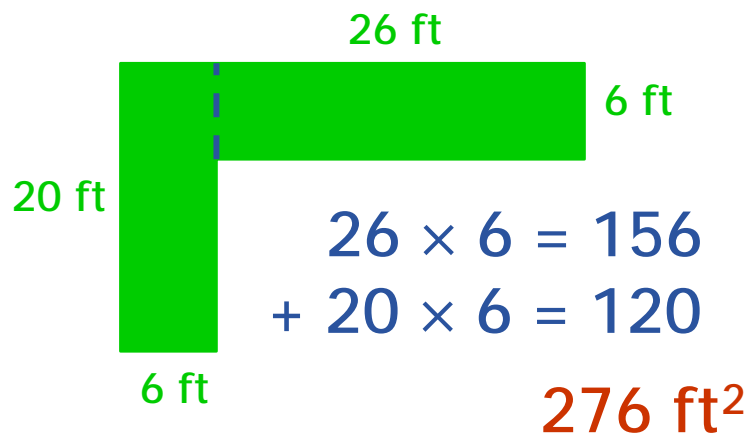
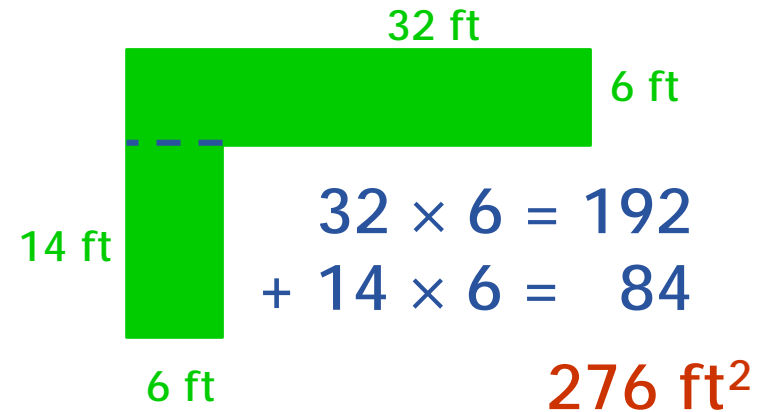
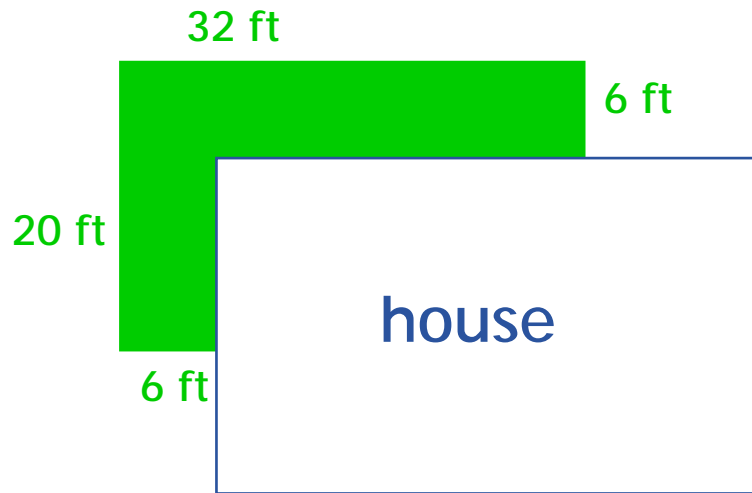
Most Missed Questions: Geometry and Measurement

Area by Partitioning

- An L-shaped flower garden is shown by the shaded area in the diagram. All intersecting segments are perpendicular.

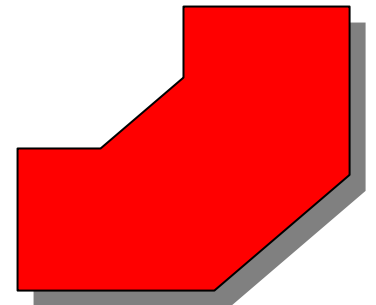
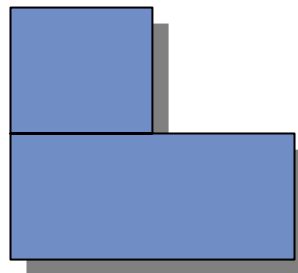
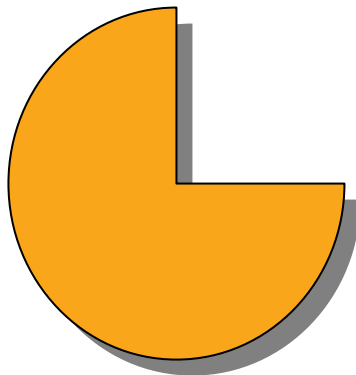


Most Missed Questions: Geometry and Measurement



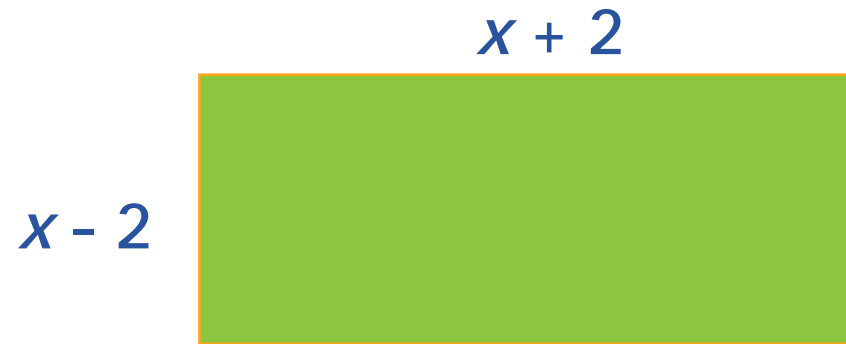
Measuring Irregular Figures

They're Everywhere!



$$a^2 + b^2 = c^2$$

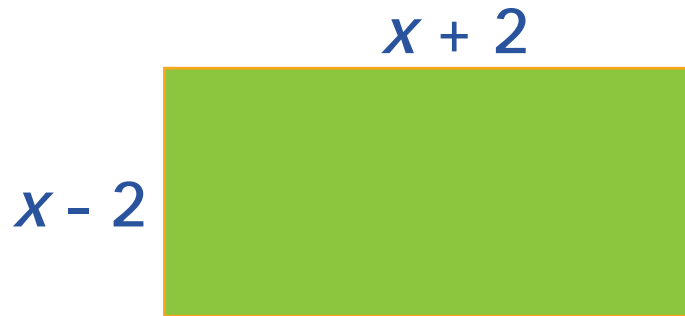
Most Missed Questions: Geometry and Measurement



Which expression represents the area of the rectangle?

- (1) $2x$
- (2) x^2
- (3) $x^2 - 4$
- (4) $x^2 + 4$
- (5) $x^2 - 4x - 4$

Most Missed Questions: Geometry and Measurement



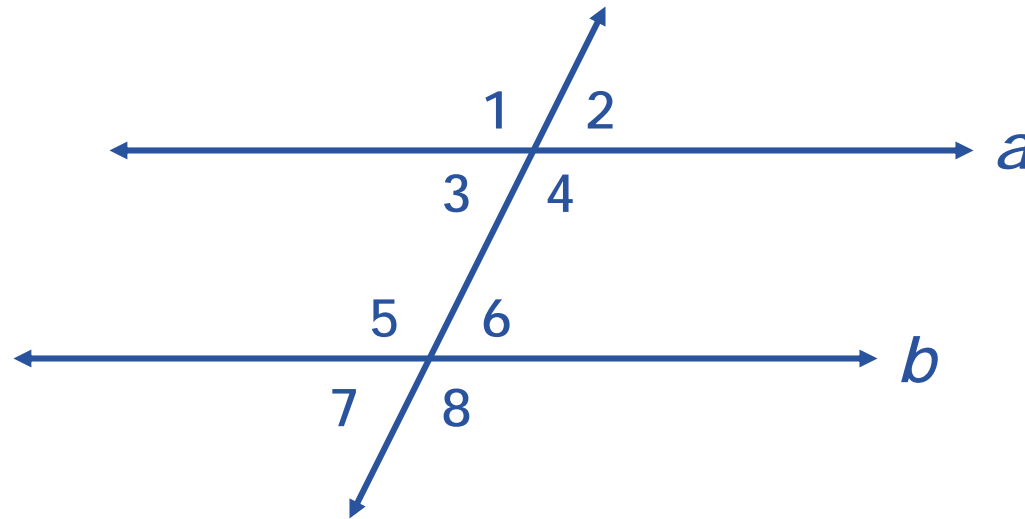
Choose a number for x .
I choose 8. Do you see any restrictions? Determine the answer numerically.

$$(8 + 2 = 10; 8 - 2 = 6; 10 \times 6 = 60)$$

Which alternative yields that value?

- | | |
|--------------------|---------------------------------------|
| (1) $2x$ | $2 \times 8 = 16$; not correct (60). |
| (2) x^2 | $8^2 = 64$; not correct. |
| (3) $x^2 - 4$ | $8^2 - 4 = 64 - 4 = 60$; correct! |
| (4) $x^2 + 4$ | $8^2 + 4 = 64 + 4 = 68$. |
| (5) $x^2 - 4x - 4$ | $8^2 - 4(8) - 4 = 64 - 32 - 4 = 28$ |

Most Missed Questions: Geometry and Measurement



Parallel Lines

- If $a \parallel b$, **ANY** pair of angles above will satisfy one of these two equations:

$$\angle x = \angle y$$

$$\angle x + \angle y = 180$$

Which one would you pick?

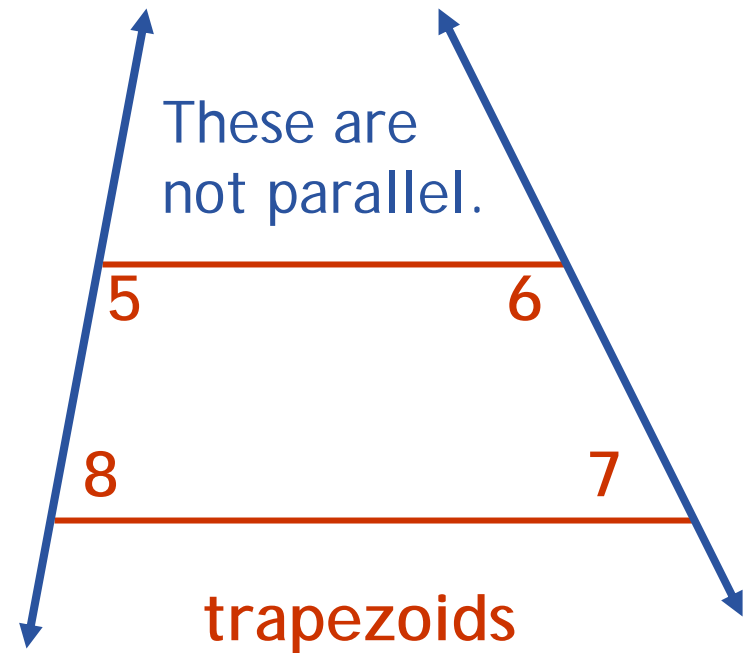
If the angles look equal (and the lines are parallel), they are!

If they don't appear to be equal, they're not!

Most Missed Questions: Geometry and Measurement



parallelograms



trapezoids

Where else are candidates likely to use the relationships among angles related to parallel lines?

Most Missed Questions: Geometry and Measurement

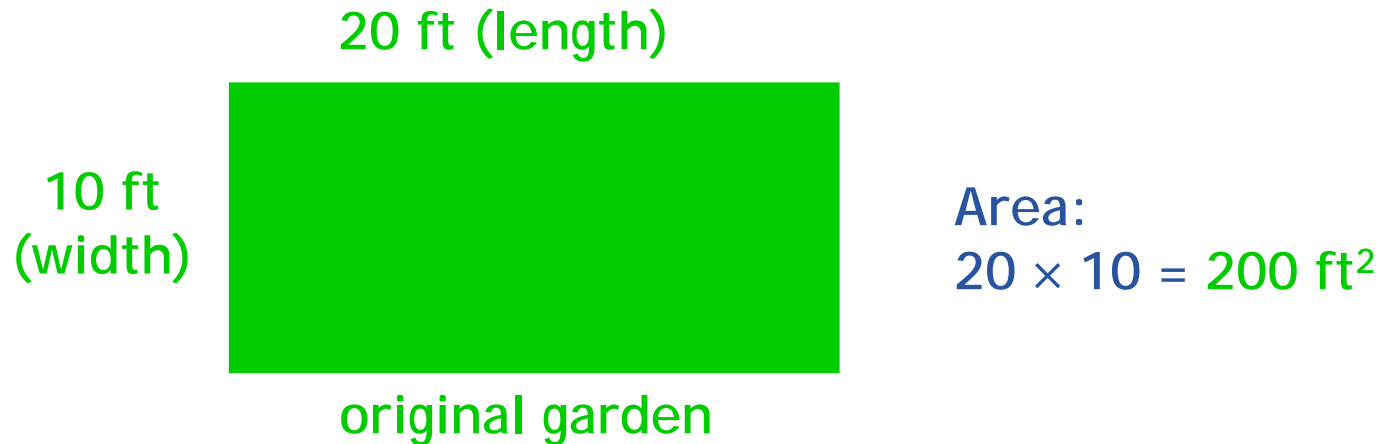
- **Comparing Areas/Perimeters/Volumes**

A rectangular garden had a length of 20 feet and a width of 10 feet. The length was increased by 50%, and the width was decreased by 50% to form a new garden. How does the area of the new garden compare to the area of the original garden?

The area of the new garden is

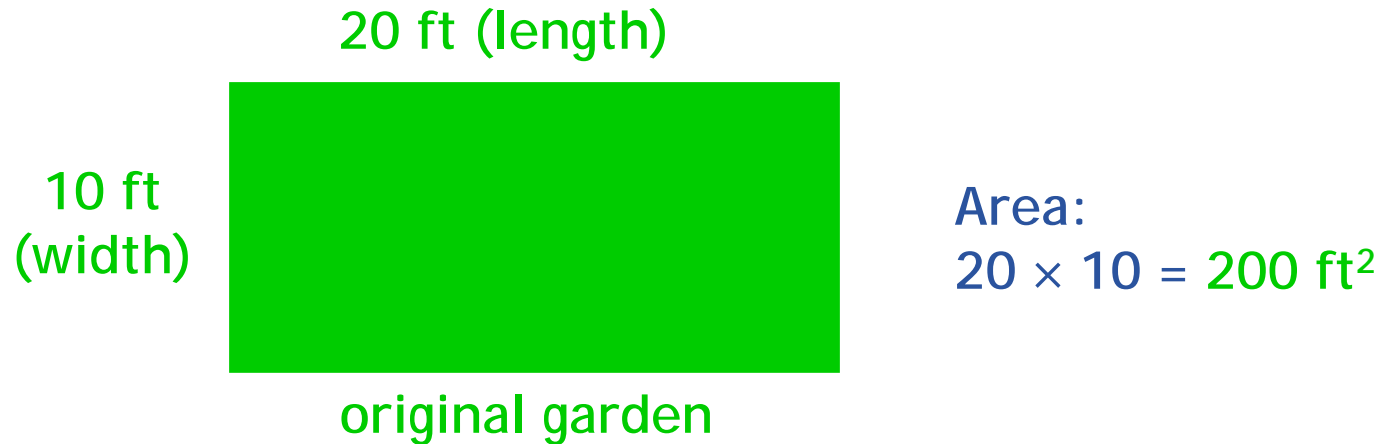
- (1) 50% less
- (2) 25% less
- (3) the same
- (4) 25% greater
- (5) 50% greater

Most Missed Questions: Geometry and Measurement



The new area is 50 ft^2 less; $50/200 = 1/4 = 25\%$ less.

Most Missed Questions: Geometry and Measurement



How do the perimeters of the above two figures compare?

What would happen if you decreased the length by 50% and increased the width by 50%

Focus on Geometry

Order by Size!

$$a^2 + b^2 = c^2$$

The Van Hiele Theory

- Level 1: Visualization
- Level 2: Analysis
- Level 3: Informal deduction
- Level 4: Formal deduction
- Level 5: Rigor

Level 1: Visualization

- Students can name and recognize shapes by their appearance, but cannot specifically identify properties of shapes. Although they may be able to recognize characteristics, they do not use them for recognition and sorting.
- Students manipulate physical models, e.g., lay one shape on top of another.

Level 2: Analysis

- Students begin to identify attributes of shapes and learn to use appropriate vocabulary related to attributes, but do not make connections between different shapes and their properties.
- In the example of the area of a rectangle, students can count the component square units.

Level 3: Informal Deduction

- Students are able to recognize relationships between and among properties of shapes or classes of shapes and are able to follow logical arguments using such properties.
- See the relationship between length, width, and area for all rectangles.

$$a^2 + b^2 = c^2$$

Level 4: Formal Deduction

- Students can go beyond just identifying characteristics of shapes and are able to construct proofs using postulates or axioms and definitions. A typical high school geometry course should be taught at this level.

$$a^2 + b^2 = c^2$$

Level 5: Rigor

- Students at this level can work in different geometric or axiomatic systems and would most likely be enrolled in a college-level course in geometry.

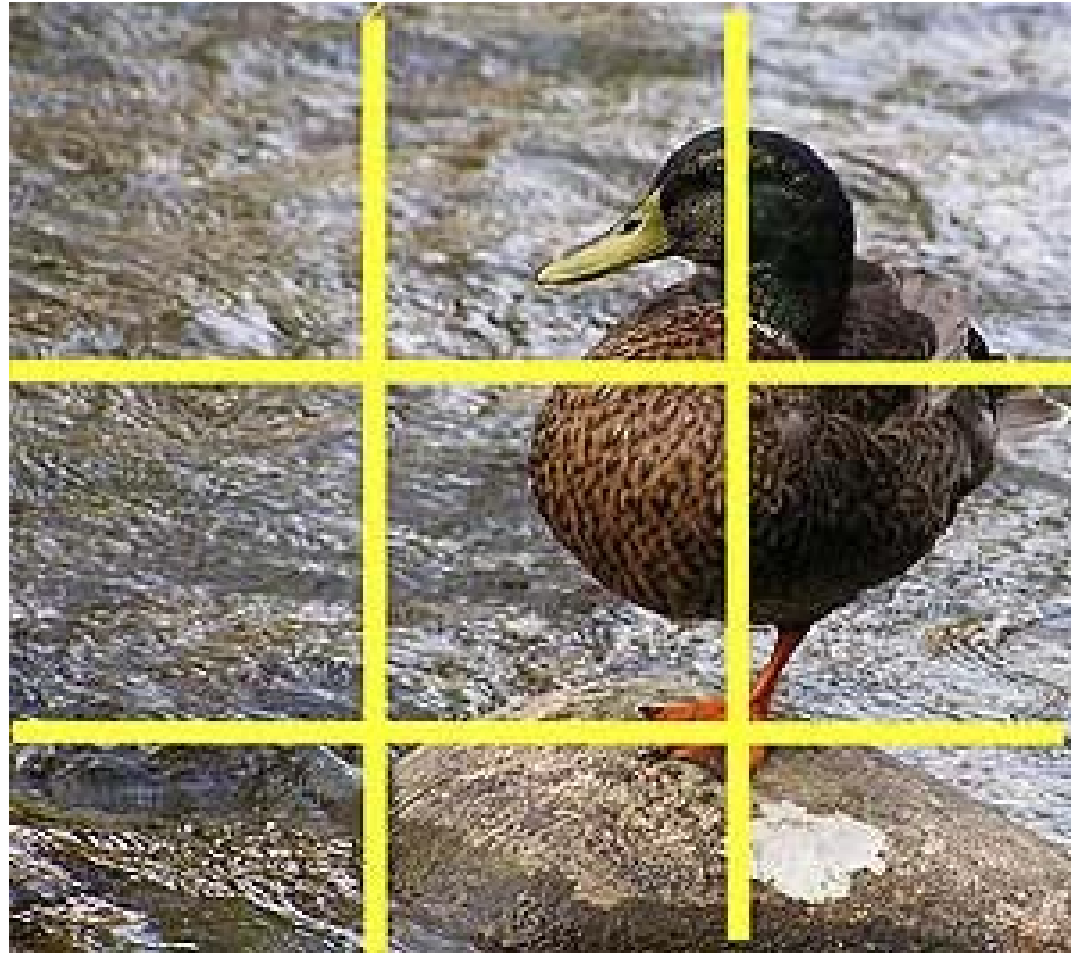
$$a^2 + b^2 = c^2$$

An Integrated Math Lesson Plan

Checking Out Da Vinci

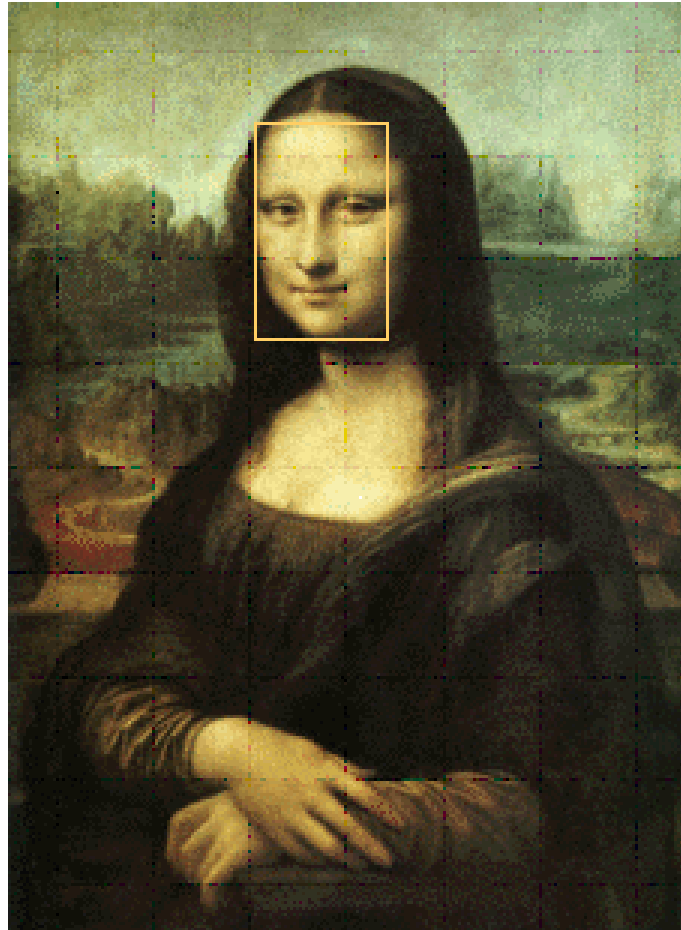
$$a^2 + b^2 = c^2$$

Checking Out DaVinci



$$a^2 + b^2 = c^2$$

Checking Out DaVinci



$$a^2 + b^2 = c^2$$

Checking Out DaVinci

The “golden rectangle” discovered by the ancient Greeks is thought to have an especially pleasing shape. The length (L) of this rectangle in terms of its width (W) is given by the following formula.

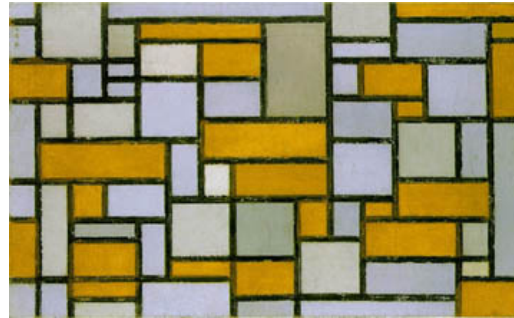
$$L = \frac{W \times (1 + \sqrt{5})}{2}$$

If the width of a golden rectangle is 10 meters, what is its approximate length in meters?

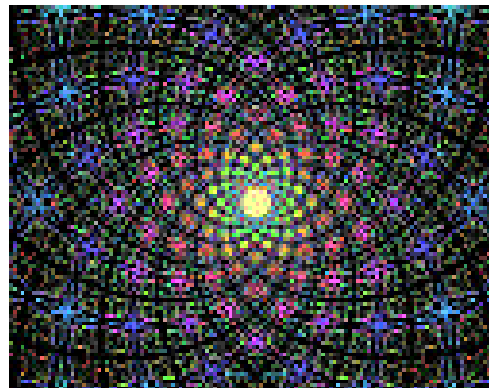
- (1) 6.1
- (2) 6.6
- (3) 11.2
- (4) 12.2
- (5) 16.2

This question may be difficult even with the calculator. Is there another way to get an idea of what the correct answer may be?

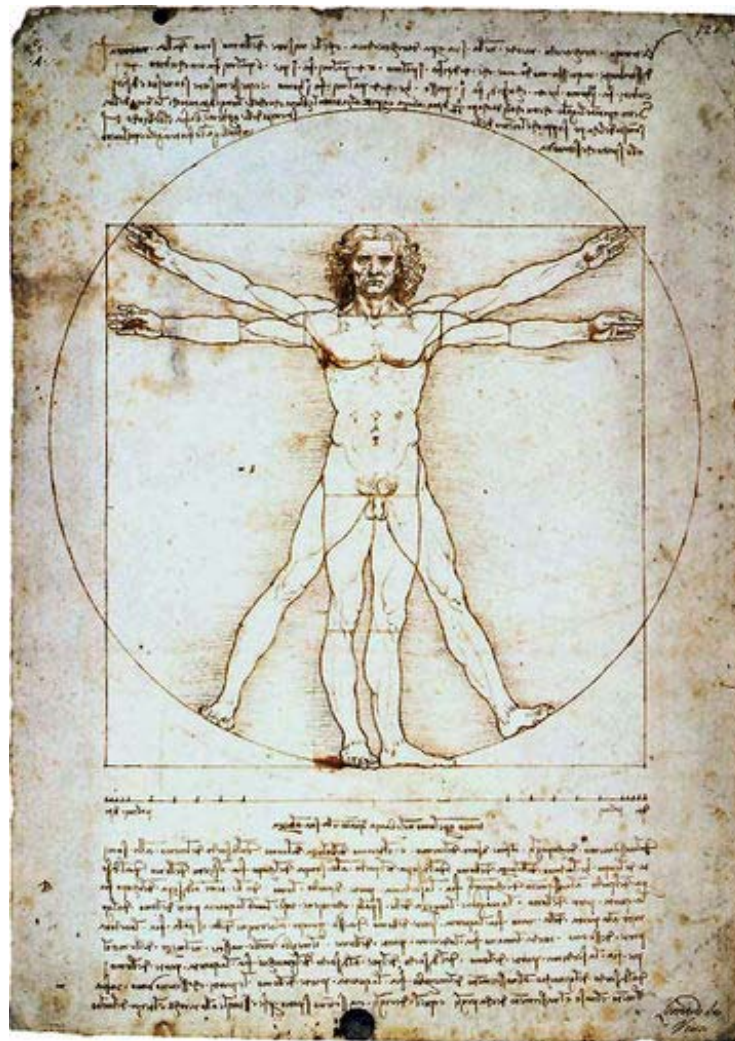
Checking Out DaVinci



GOLDEN
RECTANGLES

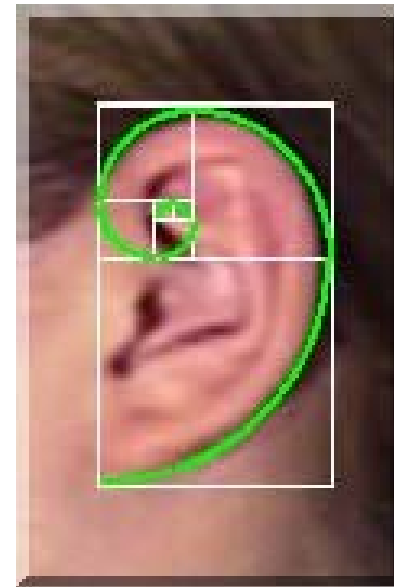
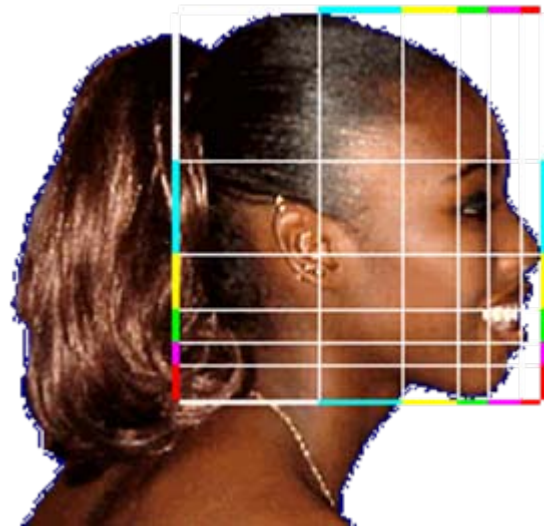


Checking Out DaVinci



$$a^2 + b^2 = c^2$$

Checking Out DaVinci



$a^2 + b^2 = c^2$

Focus on Geometry

Computer Lab Time!



Hidden Polygons

Get ready for tessellations by locating the hidden polygons on your handout. Color or shade them in on the paper.

$$a^2 + b^2 = c^2$$

Activity 1: Tessellations

http://nlvm.usu.edu/en/nav/frames_asid_169_g_1_t_3.html?open=activities

http://nlvm.usu.edu/en/nav/frames_asid_163_g_4_t_3.html?open=activities

Activity 2: Using Geoboards

http://nlvm.usu.edu/en/nav/frames_asid_277_g_1_t_3.html?open=activities

$$a^2 + b^2 = c^2$$

Activity 3: Geometry Website Tour

<http://www.k111.k12.il.us/King/math.htm#About%20Space%20Activities>

<http://mathforum.org/geometry/geom.lessons.html>

http://www.thefutureschannel.com/hands-on_math/geometry.php

$$a^2 + b^2 = c^2$$

Activity 4: Turtle Geometry!

http://nlvm.usu.edu/en/nav/frames_asid_178_g_4_t_3.html?open=activities

$$a^2 + b^2 = c^2$$

Activity 5: Explore

It's Geometry Time!

$$a^2 + b^2 = c^2$$

Focus on Geometry Websites

- *AAAMath* <http://aaamath.com>
- *Annenburg Geometry*
<http://www.learner.org/teacherslab/math/geometry/across.html>
- *Animated Pythagorean Theorem.*
<http://www.nadn.navy.mil/MathDept/mdm/pyth.html>
- *Atlas of Geometry Expressions.*
<http://www.geometryexpressions.com/GeometryAtlas/GeometryAtlas.php>

Focus on Geometry Websites

- *Brunnermath*
<http://www.brunnermath.com/geometry.htm>
- *Florida TechNet* <http://floridatechnet.org/>
- *Futures Channel*
http://www.thefutureschannel.com/hands-on_math/geometry.php
- *Geometry Center* <http://www.geom.uiuc.edu/>
- *Geometry Junkyard*
<http://www.ics.uci.edu/~eppstein/junkyard/>

Focus on Geometry Websites

- **Geometry Prep**
<http://www.regentsprep.org/Regents/math/geometry/math-GEOMETRY.htm>
- **Geometry Online**
<http://math.rice.edu/~lanius/Geom/>
- **Illuminations** <http://illuminations.nctm.org/>.
- **Introduction to Geometry**
<http://library.thinkquest.org/2647/geometry/intro/intro.htm>
- **King's List of Online Math Activities**
<http://www.k111.k12.il.us/King/math.htm#About%20Space%20Activities>
- **MathBits.com** <http://mathbits.com/>

Focus on Geometry Websites

- *Math Forum and Drexel University.*
<http://mathforum.org/>
- *National Council of Teachers of Mathematics*
<http://www.nctm.org/>
- *Pascal's Triangle.*
<http://mathforum.org/workshops/usi/pascal/>
- *PBS Teacher Source.*
<http://www.pbs.org/teachersource/math.htm>
- *Quia Top 20 Math Games*
<http://www.quia.com/dir/math/>

Focus on Geometry Websites

- *Shape and Space*
<http://www.learner.org/teacherslab/math/geometry/>
- *Teaching Strategies for Mathematics*
http://images.rbs.org/teaching_strategies/index.shtml
- *The Franklin Institute.*
<http://sln.fi.edu/tfi/hotlists/math.html>.
- *The Math Forum.*
<http://mathforum.org/>
- *The Top Ten Sites.*
http://www.exploratorium.edu/learning_studio/cool/mathematics.html

Focus on Geometry Manipulatives

- *Computing Technology for Math Excellence*
http://www.ct4me.net/math_manipulatives.htm
- *Java Interactive Geometry.*
<http://www.geom.uiuc.edu/java/>
- *Handmade Manipulatives for Math Make your own manipulatives using these templates*
<http://mason.gmu.edu/~mmankus/Handson/manipulatives.htm>
- *Manipula Math Education and Technology International Education Software*
<http://www.ies.co.jp/math/java/index.html>

Focus on Geometry Manipulatives

- *Math Templates Georgia Department of Education.*
<http://www.glc.k12.ga.us/trc/cluster.asp?mode=browse&intPathID=4985>
- *National Library of Virtual Manipulatives*
<http://nlvm.usu.edu/en/nav/vlibrary.html>
- *Shodor Math*
<http://www.shodor.org/interactivate/activities/>

Tips from GEDTS: Geometry and Measurement

- Any side of a triangle CANNOT be the sum or difference of the other two sides (Pythagorean Theorem).
- If a geometric figure is shaded, the question will ask for area; if only the outline is shown, the question will ask for perimeter (circumference).
- To find the area of a shape that is not a common geometric figure, partition the area into non-overlapping areas that are common geometric figures.
- If lines are parallel, any pair of angles will either be equal or have a sum of 180° .
- The interior angles within all triangles have a sum of 180° .
- The interior angles within a square or rectangle have a sum of 360° .

Kenn Pendleton, GEDTS Math Specialist

Focus on Geometry: Final Tips

- Candidates do not all learn in the same manner. Presenting alternate ways of approaching the solution to questions during instruction will tap more of the abilities that the candidates possess and provide increased opportunities for the candidates to be successful.
- After the full range of instruction has been covered, consider revisiting the area of geometry once again before the candidates take the test.

$$a^2 + b^2 = c^2$$

Teaching Math in the Multi-Level Classroom: Focus on Geometry

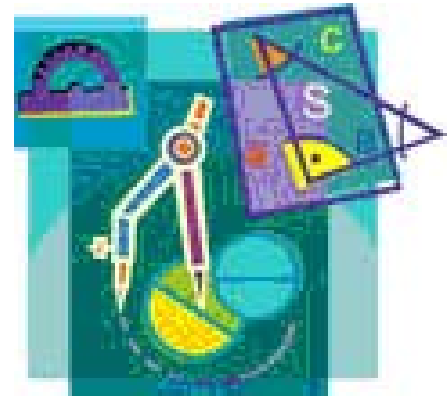
E-Learning Connections, Inc.

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$a^2 + b^2 = c^2$